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Binary pulsars as probes of relativistic gravity

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Until now, most experiments have succeeded in testing relativistic gravity only in its extreme weak-field limit. Because of the strong self-gravity of neutron stars, observations of pulsars in binary systems provide a unique opportunity for probing the strong-field régime of relativistic gravity. The two basic approaches to using binary pulsar measurements as probes of relativistic gravity are reviewed: the phenomenological ('parametrized post-keplerian' formalism) and the alternative-theory approach (multidimensional space of possible theories). The experimental constraints recently derived from the actual timing observations of three binary pulsars are summarized. General relativity passes these new, strong-field tests with complete success.

1. Introduction

The discovery of pulsars (Hewish *et al.* 1968) has had a profound impact on our view of relativistic gravity. It is true that general relativity awoke in the early sixties, out of a long dormant stage (1920–60) under the combined revitalizing influences of new experiments (starting with the one of Pound and Rebka in 1960), the discovery of new astrophysical objects (quasars, cosmic microwave background...) and new theoretical ideas (gravitational waves, black holes...) However, the discovery of pulsars (notably after the identification of a fast pulsar at the centre of the Crab nebula) brought the first experimental evidence for the existence of astrophysical objects (neutron stars) which needed a relativistic theory of gravity for their description. When one compares the (relativistic) surface gravitational potential of a $1.4M_{\odot}$ neutron star $(GM_{\rm NS}/c^2R_{\rm NS} \approx 0.2)$ with those of an ordinary star $(GM_{\odot}/c^2R_{\odot} \approx 2\times10^{-6})$ and of a black hole $(GM_{\rm BH}/c^2R_{\rm BH} = 0.5)$, one realizes that the observation of neutron stars is our only present handle on the strong-field régime of relativistic gravity. (Indeed, there is still no direct experimental evidence for the existence of black holes.) However, an isolated pulsar does not seem to be a very useful laboratory for studying strong-gravitational-field effects. Indeed, on the one hand the experimental data do not give access to the mass of the rotating neutron star, and on the other hand the strong-field effects will be time independent, and difficult to disentangle from the (poorly known) intrinsic features of the pulsar emission mechanism. I will not discuss here the use of isolated pulsars as probes of weak-field (Solar System) relativistic gravity and of very low frequency gravitational waves (see Taylor, this symposium).

In view of the above, we are very fortunate that Hulse & Taylor (1975) discovered the existence of 'binary pulsars', i.e. of pulsars members of gravitationally bound

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binary systems. Indeed, in these systems the orbital motion of the pulsar gives rise to a rich array of signatures in the observed times of arrival (TOAS), and thereby give us many handles on the relativistic gravitational interaction of two strongly self-gravitating bodies. (I will often assume, for definiteness, that one is considering binary pulsar systems where the unobserved companion is another neutron star. This assumption is not at all crucial for the discussion of strong-field tests below.)

Let us recall that, up to the discovery of binary pulsars, and apart from the qualitatively fascinating but quantitatively poor confirmations of general relativity coming from cosmological data, the only available testing ground for relativistic gravity was the Solar System. There, the years 1960–80 have been a period of intensive research in experimental gravity. From the experimental point of view, the success of this activity was due to the availability of new, high-precision technologies: e.g. the Mössbauer effect, radar and laser ranging to Solar System bodies, atomic clocks, etc. From the theoretical point of view, the conception and interpretation of new tests of relativistic gravity was greatly assisted by the existence of continuous classes of alternative (i.e. non-einsteinian) relativistic theories of gravitation. Indeed, as in ordinary life where you take better notice of the specific properties of an object when it is compared and contrasted with something else, the specific structure of Einstein's theory can be better understood when contrasted with alternative gravity theories. From that point of view, the possibility of embedding Einstein's theory within the theoretically well-motivated one-parameter family of scalar–tensor theories of gravitation due to Jordan (1949, 1955, 1959) Fierz (1956), and Brans & Dicke (1961) has been quite useful. Very useful also has been the development of the parametrized post-newtonian (PPN) formalism (Eddington 1922; Nordtvedt 1968; Will 1971; Will & Nordtvedt 1972). This formalism represents Einstein's theory as one particular point within a multi-dimensional space of alternative theories. Each dimension in that space (i.e. each parameter of the PPN formalism: $\gamma, \beta, \xi, \alpha_1, \alpha_2, \ldots$) represents a 'direction' in which a very generic alternative theory might differ in its weak-field predictions from general relativity. The intensive, multi-pronged effort in Solar System experimental gravity of the period 1960–80 can be summarized by saying that, within the assumptions of the PPN framework (notably the absence of any specific length scale in the gravitational interaction), the limiting régime of weak and quasi-stationary gravitational fields has been fairly completely mapped out at the first post-newtonian level (i.e. when taking into account fractional corrections of order $(v/c)^2 \approx GM/c^2R$ to a newtonian description of gravity), and found to agree with general relativity within a fractional accuracy of about 2×10^{-3} (for reviews see Will 1981, 1992). In technical terms, all the PPN parameters measuring a possible deviation from general relativity, i.e. $\gamma-1$, $\beta-1$, ξ, α_1, α_2, ..., have been found to be smaller, in absolute value, than 2×10^{-3} (the only possible exceptions concern bizarre parameters (ζ_i) that are not expected to come out of any decent theory).

In spite of their impressive quantitative value, Solar System tests have an important qualitative weakness: they say *a priori* nothing about how the 'correct' theory of gravity might behave in presence of strong gravitational fields, such as near a neutron star. And indeed, the PPN formalism has provided specific examples of theories (e.g. Rosen's bimetric theory) which coincide with general relativity in the post-newtonian limit while leading to very different predictions in the strong-field and/or rapidly varying-field régimes (Will 1981). (The tensor–multiscalar theories discussed below provide infinitely many other examples of such theories.) In view of this situation, it is important to assess, in a detailed manner, to what extent the observation of binary pulsars can probe new régimes of relativistic gravity left unprobed by Solar System experiments.

Immediately after the discovery of the first binary pulsar PSR 1913+16 (Hulse & Taylor 1975), it was pointed out by several authors (Damour & Ruffini 1974; Esposito & Harrison 1975; Barker & O'Connell 1975; Hari Dass & Radhakrishnan 1975; Zel'dovich & Shakura 1975; Smarr & Blandford 1976) that the 'magnetic' aspects of gravity might be detectable in the observation of binary pulsars ('spin–orbit coupling'). (Note that, although one can rightfully argue that 'gravitomagnetism' has been indirectly probed in several Solar System experiments, it seems important to have a more direct evidence of its reality, especially in the strong-field context of a spinning neutron star.) Actually PSR 1913+16 has been somewhat disappointing on this account (see, however, Weisberg *et al.* 1989), but the analysis summarized below show that PSR 1534+12 should soon fulfil our expectations. Still at the time of the discovery of PSR 1913+16, it was also pointed out (Esposito & Harrison 1975; Wagoner 1975) that gravitational radiation reaction effects, though extremely small (about 10^{-14} times the main $1/R^2$ gravitational interaction), should become observable as they accumulate with time. As is by now well known, the effect of gravitational radiation reaction on the secular change of the orbital period has indeed been observed by Taylor and collaborators (see Taylor & Weisberg 1989; Taylor 1992, and references therein). The report of this observation in December 1978 (Taylor *et al.* 1979) has spurred a lot of theoretical work on gravitational radiation reaction effects in gravitationally bound systems (for reviews see Will 1986; Damour 1987). Indeed, after the announcement that these effects have been seen, many theorists realized that the theoretical formula due to Peters & Mathews (1963) used to estimate the effect had been derived only heuristically, and, moreover, only for weakly self-gravitating systems. In the end, more rigorous methods (especially an exhaustive analysis of the general relativistic dynamics of binary systems of strongly self-gravitating bodies (Damour & Deruelle 1981; Damour 1982, 1983)) succeeded in deriving the general relativistic prediction for the orbital period change $\dot{P}_b$ in binary systems of neutron stars. This prediction has the form of an explicit mathematical formula relating $\dot{P}_b$ to the orbital period P_b, the eccentricity e and the inertial masses of the pulsar and its companion, m_1 and m_2. Introducing the notation

$$M \equiv m_1+m_2, \quad X_1 \equiv m_1/M, \quad X_2 \equiv m_2/M \equiv 1-X_1,$$
$$n \equiv 2\pi/P_b, \quad P_4(e) \equiv 1+\tfrac{73}{24}e^2+\tfrac{37}{96}e^4,$$

this formula reads

$$\dot{P}_b^{GR}(m_1, m_2) = -(192\pi/5c^5)\,X_1X_2(GMn)^{\frac{5}{3}}P_4(e)/(1-e^2)^{\frac{7}{2}}. \tag{1.1}$$

The result (1.1) formally coincides with the previously derived heuristic one, but, thanks to the new derivations, one has learned a lot about how strong-field effects in general relativity get renormalized away in the definition of the inertial masses m_1 and m_2 (these masses include self-gravity contributions which amount to about 15%, i.e. 30 times the present accuracy in the measurement of $\dot{P}_b$!).

To compare the theoretical prediction (1.1) with the observed orbital period change $\dot{P}_b^{obs}$ one needs two other pieces of information enabling one to compute the values of the two masses m_1 and m_2. (Moreover, the precision on $\dot{P}_b^{obs}$ has become so good that it is now necessary to correct for the small combined effect of Galactic acceleration and proper motion on the observable period change (Damour & Taylor

1991).) Fortunately, the fitting of the arrival times of PSR 1913+16 to a phenomenological, i.e. theory-independent, timing model (BT+, introduced in Damour & Taylor (1992) as an upgraded form of the original model of Blandford & Teukolsky (1976), see below) allows one to extract from the pulsar measurements, besides the expected 'Keplerian' parameters (notably, P_b, e, and the projected semi-major axis of the pulsar orbit $x \equiv a_1 \sin i/c$), three 'post-keplerian' parameters: the secular change of the orbital period $\dot{P}_b$, but also the secular advance of the periastron $\dot{\omega}$, and a time dilation parameter γ (not to be confused with the PPN parameter denoted by the same letter). The general relativistic predictions for $\dot{\omega}$ and γ as functions of the keplerian parameters $n \equiv 2\pi/P_b$ and e, and of the masses read

$$\dot{\omega}^{GR}(m_1, m_2) = (3n/(1-e^2))\,(GMn/c^3)^{\frac{2}{3}}, \tag{1.2}$$

$$\gamma^{GR}(m_1, m_2) = (e/n)\,X_2(1+X_2)\,(GMn/c^3)^{\frac{2}{3}}. \tag{1.3}$$

In graphical terms, the simultaneous measurement of the three post-keplerian parameters $\dot{\omega}^{obs}$, γ^{obs} and $\dot{P}_b^{obs}$ defines, when interpreted within the framework of general relativity, three curves in the m_1, m_2 plane, defined by the equations

$$\dot{\omega}^{GR}(m_1, m_2) = \dot{\omega}^{obs}, \quad \gamma^{GR}(m_1, m_2) = \gamma^{obs}, \quad \dot{P}_b^{GR}(m_1, m_2) = \dot{P}_b^{obs}. \tag{1.4a–c}$$

(When taking into account the finite accuracy of the measurements these curves broaden to three strips in the mass plane.) Equations (1.1)–(1.4) thereby yield one test of general relativity, according to whether the three curves meet at one point, as they should. As is discussed in detail in Taylor (this symposium), general relativity passes this test with complete success (at the accuracy level 5×10^{-3}, given by the present width of the $\dot{P}_b$ strip).

This beautiful success raises at the same time some questions. As $\dot{P}_b^{GR}$ is physically due to the radiative structure of the general relativistic gravitational interaction, one is certainly entitled to view the $\dot{\omega}-\gamma-\dot{P}_b$ test as a convincing experimental evidence for the existence of gravitational radiation. However, the rigorous derivations of $\dot{P}_b^{GR}$ show that the full strong-field structure of general relativity plays also an essential role in determining the simple (weak–field-like) formula (1.1). The same remark applies to the two other formulas (1.2) and (1.3). The validity of these remarks is clarified by using the same methodology which has proven to be so useful in the Solar System. Namely, instead of considering general relativity as an isolated theory, one contrasts it with alternative theories which differ both in their strong-field and their radiative structures. This methodology has been applied to the $\dot{\omega}-\gamma-\dot{P}_b$ test by Eardley (1975), Will & Eardley (1977) and Will (1981). In particular, it was found that Rosen's bimetric theory (which has the same post-newtonian limit as general relativity) fails the test by several orders of magnitude (besides predicting an opposite sign!) because of the interplay between strong-field and radiative effects. However, this shows also that the $\dot{\omega}-\gamma-\dot{P}_b$ test is a mixed test which combines strong-field and radiative effects in an indistinct way, so that one cannot logically conclude, when the test is satisfied, that both the specific strong-field and radiative predictions of general relativity have been independently confirmed. In fact, examples of theories have recently been constructed (Damour & Esposito-Farèse 1992) which can pass both the Solar System tests and the $\dot{\omega}-\gamma-\dot{P}_b$ test, while still differing markedly from Einstein's theory because of strong self-gravity effects in the pulsar and its companion.

The mixed nature of the $\dot{\omega}-\gamma-\dot{P}_b$ test in PSR 1913+16 raises the question to know whether it is possible to extract other tests of relativistic gravity from binary pulsar measurements, specifically tests that probe the quasi-stationary, strong-field aspects of the gravitational interaction. The possibility to extract from binary pulsar timing data other strong-field tests has been pointed out by Damour & Deruelle (1986) and Damour (1988). Recently, Damour & Taylor (1992) have tackled this problem in a more general and comprehensive manner. Among the new features of the latter work let us mention: (i) pulse-structure data are considered on par with timing data, (ii) spin-orbit and aberration effects are discussed in detail, (iii) the practical availability of the new tests is given a quantitative answer by studying the 'measurability' of the corresponding phenomenological parameters. In the following sections, I summarize the results of Damour & Taylor (1992). In particular, I follow them in distinguishing carefully the two basic possible approaches to analysing pulsar data: a phenomenological approach (PPK formalism; §2 below) or a theory-dependent approach (§3 below). Finally, Taylor *et al.* (1992) have recently applied the methodology of Damour & Taylor (1992) to actual binary pulsar data, and have succeeded in extracting from the timing data of the newly discovered binary pulsar PSR 1534+12 (Wolszczan 1991) two new tests of the quasi-stationary strong-field régime of relativistic gravity, without mixing of radiative effects. I outline their results in §4 below.

2. Phenomenological approach to binary pulsar tests (PPK formalism)

(*a*) *Phenomenological analysis of timing data*

The presentation of the $\dot{\omega}-\gamma-\dot{P}_b$ test of Einstein's theory given in equations (1.1)–(1.4) above shows clearly that this test is done in two separate steps. First, one extracts the observable parameters $\dot{\omega}^{obs}$, γ^{obs} and $\dot{P}_b^{obs}$ from the timing data. Second, one interprets these three separate measurements within the framework of a specific theory of gravitation, which predicts some explicit formulas relating the observable parameters to the (*a priori* unknown) inertial masses m_1 and m_2. To be able to separate these two steps it is necessary to dispose of a multi-parameter phenomenological (i.e. theory-independent) model, ready to be (least-squares) fitted to the actual data. Soon after the discovery of PSR 1913+16, Blandford & Teukolsky (1976) derived a phenomenological model for the timing data. The phenomenological parameters entering this 'BT' model were: on the one hand, some 'keplerian' parameters, notably the orbital period P_b, the eccentricity e, the argument of the periastron ω, and the projected semi-major axis of the pulsar orbit $x \equiv a_1 \sin i/c$, and on the other hand, some extra, 'post-keplerian' (PK) parameters: a time dilation parameter γ and various parameters representing possible secular drifts of the main orbital parameters: $\dot{P}_b$, $\dot{e}$, $\dot{\omega}$ and $\dot{x}$. Although Blandford & Teukolsky had in mind only to describe within some approximation the general relativistic dynamics of a two-body system, later work (Eardley 1975; Will 1981) showed that the BT model was equally apt at describing the timing data in a very wide class of alternative relativistic gravity theories (e.g. when considering the predictions of more general theories within the approximation of the BT model there are new physical effects contributing to the observable parameter γ, but there is no need to introduce new phenomenological parameters). However, the increased precision of the observational data obtained by Taylor and co-workers motivated several theorists to improve the timing model by including all $O(v^2/c^2)$ fractional contributions to the timing formula arising either from the gravitational time delay effects caused by the companion, from relativistic v^2/c^2 effects in the orbital motion

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of the pulsar, or from aberration effects. Initial attempts to do so (Smarr & Blandford 1976; Epstein 1977, 1979; Haugan 1985) were unsatisfactory because they were either incomplete, incorrect or because they had been derived only within Einstein's theory, so that they could not be used as phenomenological models able to test the whole spectrum of relativistic gravitation theories. By contrast, Damour & Deruelle (1985, 1986) proved that it is possible to describe all of the independent $O(v^2/c^2)$ timing effects in a simple mathematical way common to a wide class of alternative theories. This made it possible to construct a theory-independent, phenomenological timing model at the $O(v^2/c^2)$ level.

The part of the Damour–Deruelle phenomenological timing model describing orbital effects reads

$$t_b - t_0 = F[T; \{p^{K}\}; \{p^{PK}]; \{q^{PK}\}], \tag{2.1a}$$

where t_b denotes the Solar System barycentric (infinite frequency) arrival time, T the pulsar proper time (corrected for aberration),

$$\{p^{K}\} = \{P_b, T_0, e_0, \omega_0, x_0\} \tag{2.1b}$$

is the set of keplerian parameters,

$$\{p^{PK}\} = \{k, \gamma, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\} \tag{2.1c}$$

the set of separately measurable post-keplerian parameters, and

$$\{q^{PK}\} = \{\delta_r, A, B, D\} \tag{2.1d}$$

the set of not separately measurable post-keplerian parameters. The right-hand side of (2.1a) is given by

$$F(T) = D^{-1}[T + \Delta_R(T) + \Delta_E(T) + \Delta_S(T) + \Delta_A(T)], \tag{2.2a}$$

$$\Delta_R = x \sin\omega\,[\cos u - e(1+\delta_r)] + x\,[1 - e^2(1+\delta_\theta)^2]^{\frac{1}{2}} \cos\omega \sin u, \tag{2.2b}$$

$$\Delta_E = \gamma \sin u, \tag{2.2c}$$

$$\Delta_S = -2r \ln\{1 - e\cos u - s\,[\sin\omega\,(\cos u - e) + (1-e^2)^{\frac{1}{2}} \cos\omega \sin u]\}, \tag{2.2d}$$

$$\Delta_A = A\,\{\sin[\omega + A_e(u)] + e\sin\omega\} + B\,\{\cos[\omega + A_e(u)] + e\cos\omega\}, \tag{2.2e}$$

where

$$x = x_0 + \dot{x}(T - T_0), \quad e = e_0 + \dot{e}(T - T_0), \tag{2.3a, b}$$

and where $A_e(u)$ and ω are the following functions of u,

$$A_e(u) = 2\arctan\,[((1+e)/(1-e))^{\frac{1}{2}} \tan\tfrac{1}{2}u], \tag{2.3c}$$

$$\omega = \omega_0 + kA_e(u), \tag{2.3d}$$

and u is the function of T defined by solving the Kepler equation

$$u - e\sin u = 2\pi[((T-T_0)/P_b) - \tfrac{1}{2}\dot{P}_b((T-T_0)/P_b)^2]. \tag{2.3e}$$

The DD model reduces to the BT one after setting $\delta_r = \delta_\theta = r = A = B = 0$, $D = 1$, and replacing $\omega_{DD}(T)$, defined by (2.3c–e), by the simple linear function $\omega_{BT} = \omega_0 + \dot{\omega}(T - T_0)$. After these replacements the DD parameter k can be simply identified with the BT parameter $\dot{\omega}P_b/2\pi$. More complete descriptions of the DD timing model are given in Damour & Deruelle (1986), Taylor & Weisberg (1989) and Damour & Taylor (1992).

Although the splitting of $F(T)$ into the various contributions (2.2b–e) is a coordinate-dependent concept, it is convenient to refer to Δ_R, the time of flight across the orbit, as the 'Roemer time delay', to Δ_E as the 'Einstein time delay', to Δ_S as the 'Shapiro time delay', and to Δ_A as the 'aberration delay', or difference between the actual proper time of emission and the corresponding time if the pulsar mechanism had been, say, a radial pulsation instead of a rotating beacon. In this language δ_r and δ_θ quantify relativistic ($O(v^2/c^2)$) deformations of the orbit, k describes both the secular precession and the short-period 'nutation' of the argument of the periastron, r and $s \equiv \sin i$ measure the 'range' and 'shape' of the Shapiro delay, and A and B parameterize the effects of aberration on pulse timing.

Note that the DD model differs from the BT one in two ways: it introduces new parameters corresponding to effects not included in BT, but it also associates more effects with 'old' parameters than BT does. Such is notably the case for the parameter associated with the periastron advance, denoted $\dot{\omega}$ in BT and k in DD (with $\dot{\omega} = kn = 2\pi k/P_b$). In the BT model, $\dot{\omega}$ takes into account only the secular drift of the argument of the periastron, whereas in the DD model k describes also the periodic oscillations of the periastron around its linear drift. This difference motivated Damour & Taylor (1992) to introduce an upgraded version of the BT model, called BT+, which does not contain more parameters than BT, but which associates with $\dot{\omega}$ the full secular-plus-periodic effects of periastron advance. Fig. 2*a* of Damour & Taylor (1992) shows that, in the case of PSR 1913+16, the BT+ model differs by less than 1 μs from the DD one. This explains why the present timing accuracy for PSR 1913+16 (about 14 μs) allows one to extract only the three observables $\dot{\omega}$, γ and $\dot{P}_b$, and none of the other PK parameters of the DD model, such as r, s and δ_θ. (Luckily, the situation is entirely different for PSR 1534+12, as shown in fig. 2*b* of Damour & Taylor (1922). See below.)

It was shown in Damour & Deruelle (1986) that the PK parameters in the set (2.1*d*) cannot be measured separately from those in the sets (2.1*b*, *c*) because they can be completely absorbed into suitable redefinitions of the other parameters. Therefore, in the fitting process, one should set the parameters (2.1*d*) to some fiducial values, and solve only for the remaining parameters. As discussed in detail in Damour & Taylor (1992), the redefinitions of the other parameters mean that several physical effects that cause time variations of the parameters (2.1*d*), can finally be detected only through the indirect time variation they cause in the observable, solved-for parameters (2.1*c*). (For example, the spin-orbit coupling causes changes in the aberration parameters A and B, which are indirectly reflected in secular variations of x^{obs} and e^{obs}.)

Summarizing the results of a phenomenological analysis of pulsar timing data, we see that by fitting the arrival times of a binary pulsar to the DD model (2.1)–(2.3) (in which one sets the parameters (2.1*d*) to some fiducial values), one can, in principle, measure the Keplerian parameters (2.1*b*) and eight PK parameters (2.1*c*). The important point is that the eight PK parameters can be measured in a phenomenological manner, independently of the choice of a specific theory of gravity. Within the framework of any relativistic theory, each of the eight PK parameters will be expressible as a theory-dependent function of the dynamical keplerian parameters P_b, e_0, and x_0, the two unknown inertial masses, m_1 and m_2, and in some cases the polar angles, λ and η, of the spin axis of the pulsar. The problems posed by the latter dependence will be treated below, after discussion of the pulse structure parameters. (In alternative theories one will also have to assume an equation of state for the neutron-star matter. Such an assumption is unnecessary for most of the parameters in general relativity, because of its 'effacement' properties.)

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We have given in equations (1.1)–(1.3) above the theoretical predictions for $\dot{\omega}$, γ and $\dot{P}_b$ within the framework of Einstein's theory. One expects that, in different theories of gravity, the functions

$$p_i^{\mathrm{PK}} = f_i^{\mathrm{theory}}(m_1, m_2; \lambda, \eta; P_b, e_0, x_0; \text{equation of state}) \tag{2.4}$$

will differ markedly because of the strong-field effects linked with the pulsar and its companion (recall the $Gm/c^2R \approx 0.2$ for a $1.4M_\odot$ neutron star). In Damour & Taylor (1992) were presented explicit formulas allowing one to compute the functions f_i^{theory} in a wide class of theories.

Measurement of the keplerian plus n post-keplerian parameters will determine (when the polar angles λ and η do not enter the functions (2.4)) n curves in the two-dimensional mass plane whose shape and position depend strongly on the theory of gravity being used. If the theory is 'correct' (and if the binary system is 'clean', i.e. accurately represented by a simple theoretical model), the n curves should all meet at one point. Thus the measurement of n post-keplerian timing parameters yields $n-2$ tests of relativistic gravity, and, more generally, of the other ingredients of the theoretical model of the system. We therefore conclude that in the most favourable circumstances, binary pulsar timing data can provide up to $8-2=6$ tests of relativistic gravity.

(b) *Phenomenological analysis of pulse structure*

The structure of pulsar signals (intensity, pulse shape, linear polarization...) and its variation with time provides a wealth of information about physical conditions in pulsar magnetospheres and the nature of the radio emission mechanism. For binary pulsars, pulse structure data can also contain information about gravitational physics, because of interplays between the orbital motion and the gyroscopic nature of the observed periodicity. In the latter category one example that was recognized rather early was the possibility of detecting, through a secular change of pulse shape, the relativistic precession of the spin axis of PSR 1913+16 because of spin-orbit coupling (Damour & Ruffini 1974; Esposito & Harrison 1975; Hari Dass & Radhakrishnan 1975). Another source of potentially measurable effects on pulse structure is the aberration caused by orbital motion of the pulsar, which offers the possibility of measuring several otherwise inaccessible parameters (Smarr & Blandford 1976; Damour & Deruelle 1986). Damour & Taylor (1992) have recently generalized previous work on these topics by working out a general phenomenological 'pulse structure model' which takes into account all the $O(v/c)$ fractional effects of the orbital motion on the observed flux density $S(\nu, \phi)$, and observed linear polarization angle $\psi(\phi)$ as functions of the pulsar phase ϕ. The latter quantity is the angle measuring the continuous rotation of the pulsar (after 'correcting' for the aberration.) It is related to the pulsar proper time T of equations (2.1)–(2.3) through

$$\phi(T)/2\pi = \nu_p T + \tfrac{1}{2}\dot{\nu}_p T^2 + \tfrac{1}{6}\ddot{\nu}_p T^3, \tag{2.5}$$

where $\nu_p = 1/P_p$.

The 'DT pulse-structure model' has the following structure

$$S_{\mathrm{obs}}(\nu_{\mathrm{obs}}, \phi) = G[\phi; \{p^{\mathrm{K}}\}; \{\tilde{p}^{\mathrm{PK}}\}], \tag{2.6a}$$

$$\psi(\phi) = H[\phi; \{p^{\mathrm{K}}\}; \{\tilde{p}^{\mathrm{PK}}\}], \tag{2.6b}$$

where $\{\tilde{p}^{\mathrm{PK}}\}$ denotes a new set of 11 'post-keplerian' parameters, extractable in principle from pulse structure data:

$$\{\tilde{p}^{\mathrm{PK}}\} = \{\lambda, \dot{\lambda}, \kappa, \dot{\kappa}, \sigma, \dot{\sigma}, \dot{\psi}_0, \kappa', \dot{\kappa}', \sigma', \dot{\sigma}'\}, \tag{2.7}$$

where λ is the colatitude of the pulsar spin axis $\boldsymbol{s}_1$ with respect to the triad $(\boldsymbol{I}, \boldsymbol{J}, \boldsymbol{K})$ (where $\boldsymbol{K}$ is the direction from the Earth to the pulsar, and $\boldsymbol{I}$ the direction of the ascending node), i.e.

$$\boldsymbol{s}_1 = \sin\lambda\cos\eta\boldsymbol{I} + \sin\lambda\sin\eta\boldsymbol{J} + \cos\lambda\boldsymbol{K}, \tag{2.8}$$

and where I have introduced the following notation for the parameters appearing directly in the measurable effects

$$\kappa \equiv (\sin i)^{-1}\cos\eta, \quad \sigma \equiv \operatorname{cotan} i \sin\eta, \tag{2.9a, b}$$

$$\kappa' \equiv \operatorname{cotan}\lambda \operatorname{cotan} i\cos\eta, \quad \sigma' \equiv \operatorname{cotan}\lambda\,(\sin i)^{-1}\sin\eta. \tag{2.10a, b}$$

Moreover, the secular changes of the combinations (2.9), (2.10) appear in (2.7), as well as the secular change of the polarization angle at the pulse centre, ψ_0. The explicit formulas (2.6) will be found in Damour & Taylor (1992). Note that, contrarily to the situation above for the timing model, the pulse-structure model depends on the choice of a specific emission model for the pulsar in its rest frame (especially for what concerns the polarization model (2.6*b*); the intensity model (2.6*a*) requires only weaker assumptions, e.g. axial symmetry of the intrinsic emission around some 'magnetic' axis).

Summarizing, both the timing and the pulse-structure measurements of binary pulsars can be analysed in a phenomenological way, independently from the choice of a specific relativistic theory of gravitation. This analysis is called the 'parametrized post-keplerian' (PPK) formalism, and leads to the extraction from pulsar data of up to 19 fitted 'post-keplerian' parameters (besides the usual keplerian ones). Eight of them (the set (2.1*c*)) can be extracted from timing data, and the remaining eleven (the set (2.7)) from pulse-structure data (under the assumption of some pulsar emission model). These 19 phenomenological measurements represent 15 possible tests of relativistic gravity, in which strong-field effects play an important role. (Here $15 = 19-2-2$, where, as exhibited in equation (2.4) the first subtraction accounts for the *a priori* unknown masses m_1, m_2, while the second subtraction accounts for the *a priori* unknown polar angles λ, η.)

3. Alternative-theory approach to strong-field tests

Having in hands the results of the PPK analysis, it is natural to ask the following questions. What is the theoretical significance of the tests obtained by combining several phenomenological parameters? What are these tests teaching us about gravity? To answer these questions, it is necessary to generalize to the strong-field régime the alternative-theory approach which proved so useful in the weak-field conditions of the Solar System. As we recalled in §1 above, the idea there was to embed Einstein's theory within a continuous space of alternative theories. Recently, Damour & Esposito-Farèse (1992) have introduced a generic class of alternative theories which are, on the one hand, simple enough for one to be able to compute their predictions in the strong-field conditions of binary pulsar systems, and, on the other hand, general enough to provide an infinite-dimensional space of possible theories, differing by strong-field effects from general relativity.

The class of theories studied by Damour & Esposito-Farèse (1992) is the class of tensor-multiscalar theories. In these theories, the gravitational interaction is mediated, besides the exchange of a usual einsteinian tensor field $g^*_{\mu\nu}$, by the exchange of an arbitrary number of scalar fields, φ^a, $a = 1, \ldots, n$. The kinetic terms of the scalar

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fields are described by an arbitrary σ-model metric, $d\sigma^2 = \gamma_{ab}(\varphi^c)\,d\varphi^a\,d\varphi^b$, while their coupling to matter is described by an arbitrary conformal factor, $A^2(\varphi^a)$, relating the (physical) 'Fierz metric' $\tilde{g}_{\mu\nu}$ (measured by laboratory clocks and rods) to the 'Einstein' one, $g^*_{\mu\nu}$, i.e. $\tilde{g}_{\mu\nu} = A^2(\varphi)\,g^*_{\mu\nu}$. More precisely, the action describing these theories reads,

$$S_{\text{tot}} = S_{g_*} + S_\varphi + S_m, \tag{3.1}$$

with

$$S_{g_*} = \frac{c^4}{4\pi G_*}\int \frac{d^4x}{c}\,\tfrac{1}{4}\sqrt{g_*}\,R_*, \tag{3.1a}$$

$$S_\varphi = -\frac{c^4}{4\pi G_*}\int \frac{d^4x}{c}\sqrt{g_*}\,(\tfrac{1}{2}g_*^{\mu\nu}\,\gamma_{ab}(\varphi^c)\,\partial_\mu\varphi^a\,\partial_\nu\varphi^b), \tag{3.1b}$$

$$S_m = S_m[\psi_m, A^2(\varphi^a)\,g^*_{\mu\nu}]. \tag{3.1c}$$

The predictions of this class of theories have been worked out in (Damour & Esposito-Farèse 1992) for four different observationally relevant régimes: (i) quasi-stationary weak fields (Solar System conditions), (ii) rapidly varying weak fields (gravitational wave experiments), (iii) quasi-stationary strong fields (motion of neutron stars), and (iv) the mixing of strong and radiative field effects showing up in the gravitational radiation of systems of compact bodies (neutron stars). To illustrate the appearance of strong-field effects in alternative theories, let us only quote their result for the secular periastron advance in the generic theories (3.1):

$$\dot{\omega}^{\text{theory}}(m_1, m_2) = \frac{3n}{1-e^2}\left(\frac{GMn}{c^3}\right)^{\frac{2}{3}}\left\{\frac{1-\frac{1}{3}(\alpha_1\,\alpha_2)}{[1+(\alpha_1\,\alpha_2)]^{\frac{1}{3}}} - \frac{X_1(\alpha_1\,\beta_2\,\alpha_1)+X_2(\alpha_2\,\beta_1\,\alpha_2)}{6[1+(\alpha_1\,\alpha_2)]^{\frac{4}{3}}}\right\}. \tag{3.2}$$

In equation (3.2), n, e, G, M, X_1 and X_2 have the same meaning as in §1 above (except that in alternative theories one must carefully distinguish the inertial masses m_1, m_2 from various other 'gravitational masses' describing the gravitational interactions of neutron stars). The new quantities $(\alpha_1\,\alpha_2)$, $(\alpha_1\,\beta_2\,\alpha_1)$, $(\alpha_2\,\beta_1\,\alpha_2)$ entering (3.2) contain all the strong-field effects of tensor-multiscalar theories. For instance, the definition of $(\alpha_1\,\alpha_2)$ is the following contraction with respect to the 'internal' scalar indices

$$(\alpha_1\,\alpha_2) = \gamma^{ab}\left[\frac{\partial \ln A}{\partial\varphi^a} + \frac{\partial \ln m_1}{\partial\varphi^a}\right]\left[\frac{\partial \ln A}{\partial\varphi^b} + \frac{\partial \ln m_2}{\partial\varphi^b}\right]. \tag{3.3}$$

The terms in gradients of the natural logarithm of the coupling function $A(\varphi^a)$ are already present in the weak-field régime, but the terms in gradients of the logarithm of the masses are numerically important only for strongly self-gravitating bodies. As discussed in Damour & Esposito-Farèse (1992) the strong-field effects entering α_1 and α_2 can be expressed in terms of some dimensionless 'compactness' parameters. The basic compactness parameter is

$$c_{\text{A}} \equiv -2\,\partial \ln m_{\text{A}}/\partial \ln G, \tag{3.4}$$

where $A = 1,\ 2$ labels the two considered bodies. The numerical value of the compactness c_{A} depends both on the value of the mass m_{A} and on the nuclear equation of state used to describe the internal structure of a neutron star. Fits to several numerically integrated neutron star models showed that one could often approximate the dependence $c_{\text{A}}(m_{\text{A}})$ by a linear function,

$$c_{\text{A}} = k m_{\text{A}}. \tag{3.5}$$

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The experimental results reported below have used a median value of $k = 0.21M_{\odot}^{-1}$ for the slope of the compactness versus the mass.

As the numerical value of the compactness for neutron stars of mass $m_A \approx 1.4M_{\odot}$ is only about 0.3, i.e. a factor 3 below the 'maximum' compactness of 1 (formally reached for black holes), it seems still meaningful to expand all the strong-field effects in powers of c_1 and c_2. (Note that 0.3 can still be rightfully referred to as being 'strong-field' when it is compared with the compactness of the Sun, $c_{\odot} \approx 4\times10^{-6}$, or of the Earth, $c_{\oplus} \approx 9\times10^{-10}$.) When doing this expansion, one finds that the strong-field effects in any observable quantity can be represented as a power series in the compactnesses c_1 and c_2, whose coefficients depend on the considered tensor-multiscalar theory. This remark allowed Damour & Esposito-Farèse (1992) to generalize the (weak-field) PPN formalism to the strong-field conditions of binary pulsar measurements. Indeed, when computing the coefficients in the compactness-expansions of all the quantities like $(\alpha_1\alpha_2)$, $(\alpha_1\beta_2\alpha_1)$, etc., which enter the predictions of generic tensor-multiscalar theories, one finds that they form a (partly) ordered sequence of 'theory parameters', denoted

$$\gamma_1, \beta_1, \beta_2, \beta', \beta'', \beta_3, (\beta\beta')\ldots, \tag{3.6}$$

the first two parameters in the list (3.6) are equivalent to the usual PPN parameters $\gamma-1$, $\beta-1$,

$$\gamma_1 = (1-\gamma)/(1+\gamma), \quad \beta_1 = 8(\beta-1)/(1+\gamma)^2, \tag{3.7}$$

while the other ones, $\beta_2, \beta', \beta''\ldots$ represent deeper layers of structure of the relativistic gravitational interaction, left unprobed by existing Solar Systems tests. All the parameters in the list (3.6) are explicitly calculable in terms of the arbitrary functions defining a specific tensor-multiscalar theory within the class (3.1), i.e. γ_{ab}, $A(\varphi^a)$ and their gradients, e.g.

$$\gamma_1 \equiv \gamma^{\mathrm{ab}}\frac{\partial \ln A}{\partial\varphi^a}\frac{\partial \ln A}{\partial\varphi^b}.$$

(Note that all the theory parameters (3.6) are defined so as to vanish in general relativity.) In pictorial language, each parameter in the list (3.6) represents an independent direction away from Einstein's theory in an infinite-dimensional space of alternative theories of gravitation. In other words, the 'post-PPN' parameters β_2, $\beta', \beta''\ldots$ provide a chart for the yet essentially unexplored domain of strong-field effects (both in the motion and the radiation of systems of strongly self-gravitating bodies).

There are two ways in which one can combine the phenomenological (PPK) approach to binary pulsar data of §2 with the present alternative-theory approach, with its sequence of theory parameters (3.6). The first one applies to the favourable situation where it is indeed possible to extract, with good accuracy, a sufficient number of PK parameters from the raw pulsar data. In that case, one can analyse *a posteriori* the theoretical significance of the pulsar measurements by starting from the observed values of the PK parameters, and combining that information with the alternative-theory predictions such as equation (3.2). This was essentially the idea evoked above, by which the measurement of n PK parameters defines, within the framework of each specific theory, n curves in the mass-plane. This approach is illustrated in figure 8 of Damour & Taylor (1992) for the hypothetical case of the measurement of the six PK parameters $\dot\omega$, γ, $\dot P_{\mathrm{b}}$, r, s and δ_θ, and of the comparison between general relatively and one specific tensor-biscalar theory. This approach is conceptually simple, and easy to implement. However, it neglects the correlations between the various observed parameters. In the case where there are strong correlations between the various PK parameters, and where some parameters can barely be separated from the other ones, it is better to use another approach. Namely, if one considers a theory which depends on a certain (finite) number of theory parameters, one can *a priori* replace the alternative-theory predictions for the various PK parameters (e.g. equation (3.2)) in the DD model (2.1), and solve for the theory parameters, instead of the PK parameters.

4. Application to actual binary pulsar measurements

The various methodologies outlined above have been applied to actual binary pulsar measurements by Taylor *et al.* (1992). First, these authors have followed Damour & Esposito-Farèse (1992) in considering only a subsector of all the possible directions in theory-space away from general relativity. More precisely, the latter have introduced a specific two-parameter class of tensor-biscalar theories, called $T(\beta', \beta'')$, which describes in a rather generic way two yet unexplored directions in theory space, independently of the already explored directions. More precisely, the theories $T(\beta', \beta'')$ where chosen so as to coincide with Einstein's theory in the post-newtonian limit (this freezes the PPN directions, i.e. $\gamma_1 = \beta_1 = 0$), and to suppress the strong-field induced dipole radiation effects (already explored by Eardley 1975; Will & Eardley 1977; Will & Zaglauer 1989). Moreover, the restriction to the simplest case of theories containing only two scalar fields, besides a tensor one, has the effect of freezing several possible strong-field directions (namely, one has $0 = \beta_2 = \beta_3 = \ldots$). Finally, these theories explore a domain in theory space spanned by the two strong-field parameters β' and β'' in the list (3.6).

To give an idea of which strong-field deviations from Einstein's theory are explored by the theories $T(\beta', \beta'')$, one can quote the following formulas for the quantities $(\alpha_1 \alpha_2)$ and $(\alpha_1 \beta_2 \alpha_1)$ entering the new prediction (3.2) for $\dot{\omega}^{\text{theory}}$:

$$(\alpha_1 \alpha_2) = \tfrac{1}{2}\beta'(c_1^2 + c_2^2), \tag{4.1a}$$

$$(\alpha_1 \beta_2 \alpha_1) = \beta'[-c_2 + P_{2,4}(c_1, c_2)] + \beta'^2 P_{3,6}(c_1, c_2) + \tfrac{1}{2}\beta'' c_2^2. \tag{4.1b}$$

In equation (4.1 *b*) $P_{i,j}(c_1, c_2)$ denotes a polynomial in the compactnesses which starts at order i and ends at order j. By inserting (4.1) into (3.2) one sees that, because of the $(\alpha\beta\alpha)$ terms, the strong-field modifications to the general relativistic prediction (1.2) associated with β' start at order $O(c)$, while those associated with β'' start at order $O(c^2)$. For the precise definition of the theory $T(\beta', \beta'')$ and the derivation of its predictions relevant to binary pulsar measurements see Damour & Esposito-Farèse (1992) and Damour & Taylor (1992).

Besides illuminating the theoretical significance, in terms of strong-field deviations, of the various phenomenological measurements, the use of a specific family of theories like $T(\beta', \beta'')$ provides a common ground for intercomparing and combining tests coming from observations of different pulsars. In pictorial terms, the data of each pulsar define a certain allowed region (at, say, the 90% confidence level) within the space of alternative theories. Then, in the latter space, one can simultaneously represent the tests based on different pulsar data by drawing all the corresponding allowed regions. The 'correct' theory of gravity should lie in the intersection of all the allowed regions.

A pioneering analysis of the constraints imposed within the two-dimensional space of theories $T(\beta', \beta'')$ by various, presently available pulsar measurements has been recently implemented by Taylor *et al.* (1992). They made use of: (i) 10 years of high-quality timing observations of PSR 1913+16, (ii) one year of similar data for the newly discovered binary pulsar PSR 1534+12 (Wolszczan 1991), and (iii) a previously proposed theoretical interpretation (Damour & Schäfer 1991) of the keplerian parameters extracted from several years of high-quality timing observations of the 'non-relativistic' binary pulsar PSR 1855+09 (Ryba & Taylor 1991). Concerning the latter test, the idea of Damour & Schäfer (1991) was that the observation of a low-eccentricity long-period binary pulsar with a white-dwarf companion ($c_2 \ll c_1$) sets a probabilistic upper bound on any possible difference in the free-fall accelerations of a neutron star (the pulsar) and a white dwarf (the companion) in the gravitational field of our Galaxy. The theories $T(\beta', \beta'')$ predict that the ratio of the free-fall accelerations is

$$\frac{a_1}{a_2} = \frac{1+\frac{1}{2}\beta' c_1^2}{1+\frac{1}{2}\beta' c_2^2} \approx 1 + \tfrac{1}{2}\beta' c_1^2, \tag{4.2}$$

when $c_2 \ll c_1$. Therefore the $e-P_b$ PSR 1855+09 test provides an upper bound on the magnitude of β'.

Concerning the analysis of the two other sets of pulsar measurements, Taylor *et al.* (1992) used both the purely phenomenological (PPK) approach of §2, and the alternative-theory approach of §3. As already mentioned, the timing measurements of PSR 1913+16 can be very satisfactorily fitted by a simple BT+model. This means that only three PK parameters, $\dot{\omega}$, γ and $\dot{P}_b$ can be extracted, with any decent accuracy, from the presently available data (see, however, fig. 2 of Taylor *et al.* (1992) for the two further PK parameters r and s). By contrast, thanks to a more 'edge on' position, the PPK analysis of the PSR 1534+12 data allowed one to extract the four PK parameters $\dot{\omega}$, γ, r and s. As said above, these four phenomenological measurements provide $4-2=2$ new, independent tests of relativistic gravity. At present, the accuracy of these two new strong-field tests is not very high (e.g. $\sigma_r/r = 21\%$); however, numerical simulations by Damour & Taylor (1992) show that it should steadily improve as more data become available.

Shifting from the phenomenological to an alternative-theory approach, Taylor *et al.* (1992) interpreted the various pulsar data in terms of the corresponding allowed regions they define in the two-dimensional, β', β'', space of alternative theories. The PSR 1913+16 data define a thin strip (roughly located around the parabola $\beta'' = (\beta')^2$) corresponding to the single (0.5% accurate) $\dot{\omega}-\gamma-\dot{P}_b$ test. The two new (low precision) $\dot{\omega}-\gamma-r-s$ tests in PSR 1534+12 define a rather wide, potato-shaped allowed region in the β', β'' plane. Finally, the $e-P_b$ test in PSR 1855+09 corresponds to the vertical strip $-1.6 < \beta' < 1.6$ (90% confidence level). When combining these three independent allowed regions, one reaches the following conclusions. (*a*) The three allowed regions do admit a non-empty common intersection, and Einstein's theory (i.e. the point $(\beta', \beta'') = (0,0)$) lies well inside this intersection region. (*b*) At the 90% confidence level the post-PPN, strong-field parameters β' and β'' are constrained to lie in a thin parabolic segment whose projections on the β', β'' axes are roughly $-1.1 < \beta' < 1.6$, $-1 < \beta'' < 6$.

In conclusion, Einstein's theory of gravitation has passed with complete success several new, deep and sensitive experimental tests. Among these tests, the ones associated with the data from PSR 1534+12 and PSR 1855+09 concern the quasi-stationary strong-field régime of relativistic gravity, without mixing of radiative effects. Moreover, numerical simulations by Damour & Taylor (1992) show that the prospects are good for improving these tests, and extracting still new tests from future binary pulsar measurements.

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